MATHEMATICAL PROBLEM-SOLVING PERFORMANCE OF TALENTED STUDENTS IN PRIMARY GRADES IN THAILAND

***Ratchada YATRA**
ryatr@ipst.ac.th
Institute for the Promotion of Teaching Science and Technology, Thailand

Sujin SUWANNA
sujin.suw@mahidol.ac.th
Department of Physics, Faculty of Science, Mahidol University, Thailand

Surat INTASANG
surat.intasang@gmail.com
Faculty of Education, Ramkhamhaeng University, Thailand

Abstract

We have performed a quantitative and qualitative analysis of the examination results of primary school students in the Science and Mathematics Talent Development (SMTD) Project. We aim to identify students' creativity and misconceptions in mathematics among the high-achieving students. This research is conducted in conjunction with the SMTD examination administered by the Institute for the Promotion of Teaching Science and Technology (IPST). Each year more than 100,000 students nationwide take part in the examination. The mathematics examination consists of two rounds. The first round has 30-40 fill-in-the-answer problems to be completed in two hours. Statistical analysis of the examination results is analyzed to identify weaknesses of students' understanding in each strand of curriculum core (Numbers and Algebra; Measurement and Geometry; and Data Analysis and Probability) as well as geographical dependence students' achievement. Approximately the top 1,000 students are invited to take the second round of the examination, which consists of 10 problem-solving questions. Some selected questions are analyzed and our statistical, quantitative and qualitative results are shared and discussed in details.

Keywords: student-performance; talent; creativity; misconception; primary level.

Introduction

In Thailand, the Institute for the Promotion of Teaching Science and Technology (IPST) is responsible for all aspects of compulsory science and mathematics education (i.e. Grades 1-12), and it has hoped to evaluate the national science and mathematics achievement (IPST, 2012b,c, 2013a). For more than a decade, IPST has initiated the Science and Mathematics Talent Development (SMTD) Project and has administered a nationwide examination to select primary school students (Grades 1-6) into this program. With the twelve Project centres located throughout the nation, IPST is hopeful that the SMTD project will increase interest in the study science and mathematics, and that the students in the project will provide valuable research for educators to improve science and mathematics education. The Project has progressed successfully along with the training of primary-level mathematics teachers, where IPST's role is to provide educational support such as teaching media (e.g. manipulatives, books, examples of lesson plans) and teacher trainers.

The SMTD examination is administered in two rounds each year for the lower primary level (Grades 1-3) and the upper primary level (Grades 4-6). The first round of the examination typically comprises 30-40 problems to be completed in two hours, where students are asked to fill their answers (no more than three digits) in a computer-based answer sheet. A team of educators, primary school teachers, college teachers and administrators selects the examination questions, which are to cover all core curriculum strands (Numbers and Algebra; Measurement and Geometry; and Data Analysis and Probability) and learning behaviours expected from the students at these primary levels. The questions are required to fit within the scopes of teaching content and standards set by IPST under the Ministry of Education (Bureau of Academics Affairs and Educations Standards (BAAES), 2008).

In each year, approximately 100,000 students take part in the examination nationally in the first round, and their statistical results are analysed to observe the national trends and to identify weakness and strength of the students, perhaps by geography, skills, etc. From the first-round examination, about 1000-1300 students are selected to take the second-round examination, which consists of about 10 problems to be completed in two hours. The students are asked to show their work on these questions as to understand their problem-solving approaches. The second-round examination is graded with attention given to the students' explanation rather than the final answers (e.g. with weight roughly 80 to 20 percents). The students' solutions over the past four years are now being analysed to evaluate the students' problem-solving performance. In particular, we study their solutions to attempt to understand their thinking and their understanding of mathematical concepts. As such, their misconception and creativity are specifically looked for, together with how they obtain their answers and explain their reasons.

Problem Solving as a Core Competency

Only recently has IPST seriously promoted problem solving lessons in its textbooks (IPST, 2011), and has trained math teachers to impart problem-solving skills to young pupils. Over the last few years, problem solving has become one of the main modules for the SMTD project for both students and teachers (IPST, 2012a; Makanong, 2010). In other countries, such as Singapore, Australia, United Kingdom, problem-solving curriculum has been developed sooner (see Yeap, 2013, and reference therein.), and it is presumed to have strong correlation with the high national achievement in mathematics in Singapore (Yeap, 2013).

The main purpose of this research is to analyse and reflect upon students' performance on mathematical problem solving on the SMTD examination. We aim to report how Thai primary-school pupils solve open-ended mathematical problems, including their tendencies, common mistakes and misconceptions. In analysing the students' solutions to the problems, we ask ourselves whether the student has shown evidence of four main steps in problem-solving according to Polya (Polya, 1957): understanding, planning, doing and checking with the emphasis on the first three steps. We believe the second-round SMTD examination is an appropriate source to understand students' thinking. We keep in mind that this group of students are presumably above the Thai national average in mathematics achievement. For instance, any misconception by students in this group should hypothetically yield a more severe degree of misconception for the entire population of young pupils. Here we present our findings based on the 2012 second-round examination for both primary levels. In each level, we present three problems with their statements (in English translation), then results.

Lower Level Results

For the lower primary level, the selected three problems are translated into English as below. These problems represent the core strands of algebra, measurement and geometry, but they also require number and operations in order solve the problems. The questions are separated by solid lines.

Problem 8: Consider the given relationship of patterns as follows:

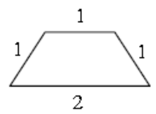
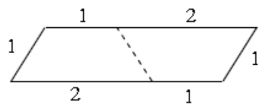
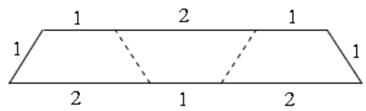
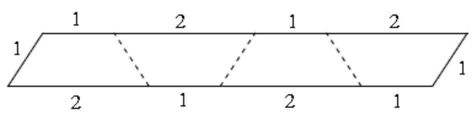
No.	Pattern	Perimeter (centimeter)
1		5
2		8
3		11
4		14

Figure 1. Illustration accompanied Problem 8 for the lower level examination.

From the above pattern, is it possible to have a figure whose perimeter is 100 centimetre long? Explain your reasons. (IPST, 2013b)

Problem 9: Do you know that the time of the day in each region on Earth is different, and it depends on the location of the region? This means that the time in different countries may be different, as shown in the figure below.

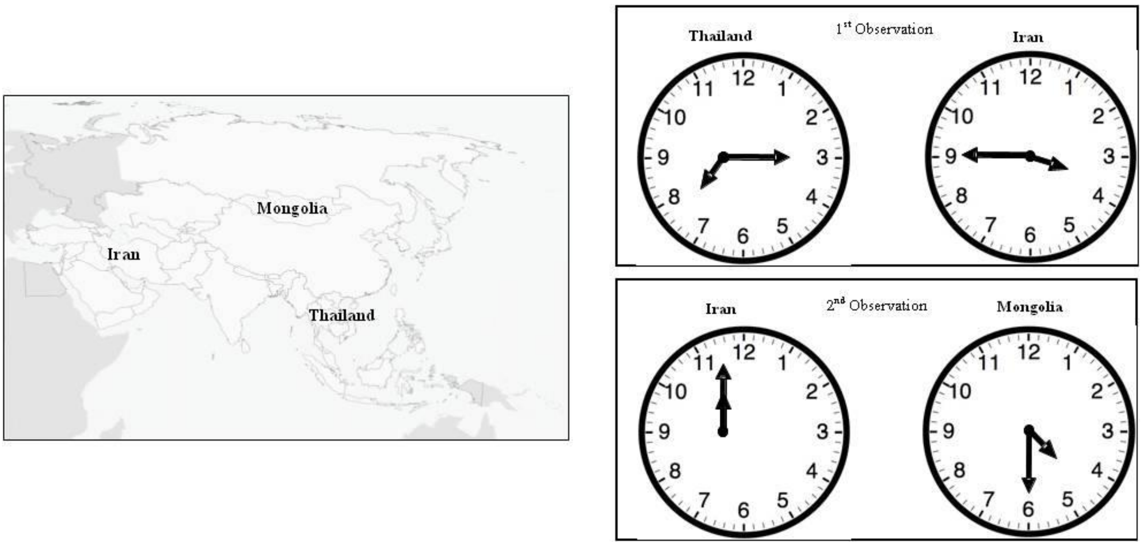


Figure 2. Pictures accompanied Problem 9 for the lower level examination.

If the clock below shows the time in Mongolia, what is the time in Thailand? Fill your answer in the box below. Draw the minute and hour hands on the clock above the box. (IPST, 2013b)

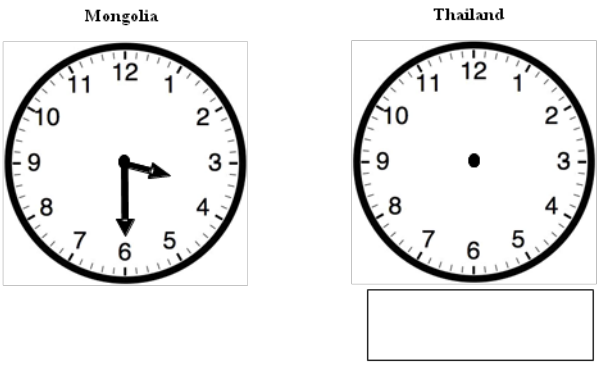
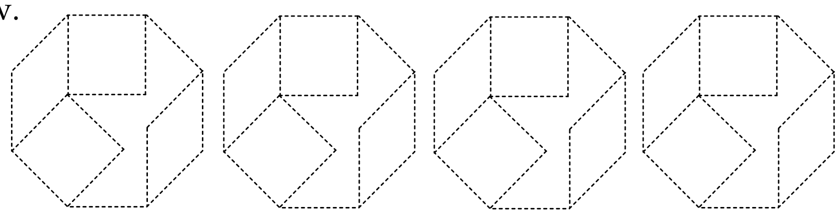


Figure 3. Pictures accompanied Problem 9 for the lower level examination where students give their answers.

Problem 10: Given the regular octagon which has lines drawn in its interior as shown in the figure below.



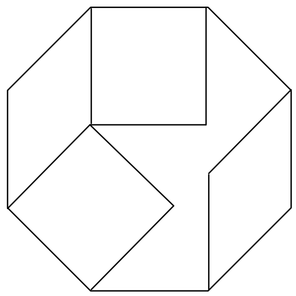


Figure 4. (left) the regular octagon given in Problem 10 for Grades 1-3; (above) same octagons (14 of them) drawn with dashed lines are provided in the answer sheet so that students can draw in their possible octagons within the regular octagon.

There are many octagons within the regular octagon given above. Find all different octagons whose sides are all equal. Draw your answers on the dashed lines in the answer sheet provided below. (IPST, 2013b)

Problems 9 and 10 are used to identify students' misconceptions in the respective concept, whereas Problem 8 is used to learn about students' creativity to solve the problem. Their statistical results are provides in tables below.

Problem 8: Result

In this problem, we divide the students into four groups and analyse their solutions. The group description and its population size are included in Table 1. Qualitatively, we observe that nearly all students who attempted to solve this problem had realized that the pattern continues by adding by 3. In fact, both groups that answer "yes" or "no" used this "adding by 3" as their reasons. For those who said "no," their common reasons were that 100 is not divisible by 3 or that 100 cannot be reached by continuing to successively add 3. Essentially, these reasons are related, but we are surprised that the students did not take the initial value (e.g. 5) into account. For those who said "yes," some say by adding by 3 the results alternate between odd and even, and therefore, 100 can be obtained. Various degrees of incorrect explanations like these result in the majority of students receiving only a few points as indicated in Table 1. In Figure 5, we show two examples of common solutions, where the incorrect solution shown on the right accounts for 36.7% of students who tried this problem.

Table 1
Distribution of Scores on Problem 8 (Grades 1-3)

	Population Percent (%)	Group Description
I	15.7	Scores 8.5 – 10 points.
II	28.6	Scores range 2.0 – 8.0 points.
III	39.8	Scores range 0.0 – 1.0 points
IV	15.9	Students do not attempt this problem.

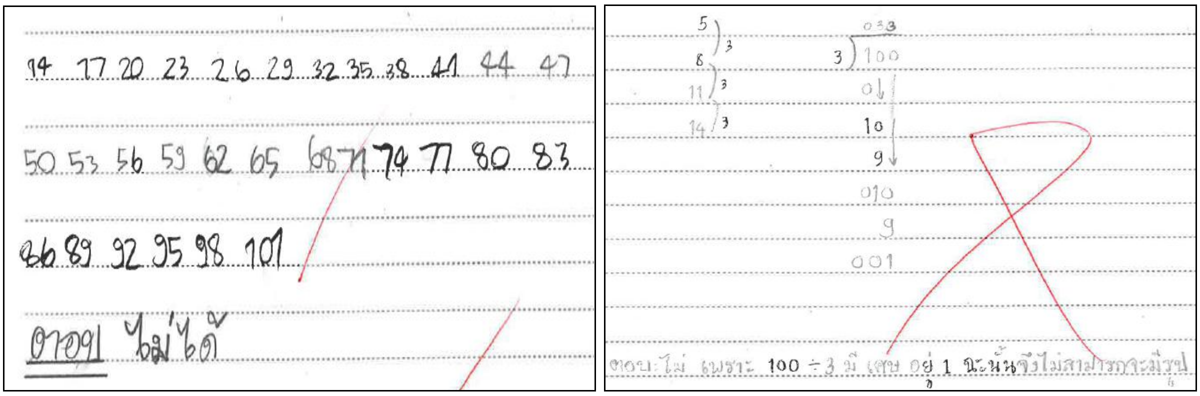


Figure 5. Examples of students' solutions on Problem 8. (left) A correct answer is obtained by simply demonstrating the number pattern up to 101, (right) whereas the majority of common incorrect reasoning is based on indivisibility by 3 of 100.

Problem 9: Result
The students are grouped into five groups according to their scores, and the statistics of their performance is shown in Table 2.

Table 2
Distribution of Scores on Problem 9 (Grades 1-3)

	Population Percent (%)	Group Description
I	4.7	Scores 10 points.
II	13.4	Scores range 6.0 – 8.0 points.
III	32.0	Scores range 1.0 – 5.0 points.
IV	40.1	Scores 0 point because of incorrect answers
V	9.8	Students do not give answers in this problem.

This problem concerns local time in different countries, and it has two correct answers: 2:30am or 2:30pm. Students can receive full credit by giving one correct answer. Among the students in Group I: 50% of them gave "2:30am," while about 39% gave "2:30pm"; thus only about 11% give both answers. But in Group II, about 54.6% gave "2:30pm." Compared with the entire population, the distribution is approximately 44%, 51.7% and 4.3%, respectively.

In Group II, it is observed that the majority of them understand the concept of time difference and are able to find the correct answers. However, sometimes their explanations are unclear or there is some minor miscalculation along the way.

In Group III, we observe that the students were not able to obtain correct answers because of miscalculation. Even though it concerns addition or subtraction only, but having modulo 12 seems to confuse them. It is conclusive, however, that their answers in the box and the drawing of the clock hands are consistent, meaning that they know how to tell time from the clock, but still have difficulty with local time calculation in different regions of the world. Similar conclusion also holds in Group IV, but students in Group IV often did not show their work or their reasoning was very far off, so they did not obtain partial credits.

Problem 10: Result
This problem does not require the students to do a calculation except counting up to 8. Yet a large number of students obtained low scores (e.g. 0–4 pts.) as shown in Table 3. This is rather surprising; and we would like to understand why.

Table 3
Distribution of Scores on Problem 10 (Grades 1-3)

Score	Population Percent (%)	Score	Population Percent (%)
0	30.1	6	4.8
1	5.9	7	5.2
2	3.5	8	8.0
3	2.5	9	6.5
4	3.8	10	14.1
5	3.9	No attempt	11.7

It is rather striking that many students do not know the phrase "octagon with equal sides." Often they gave the only one answer, namely the regular octagon; see examples in Figure 6. We are curious as to how this common misconception arises.

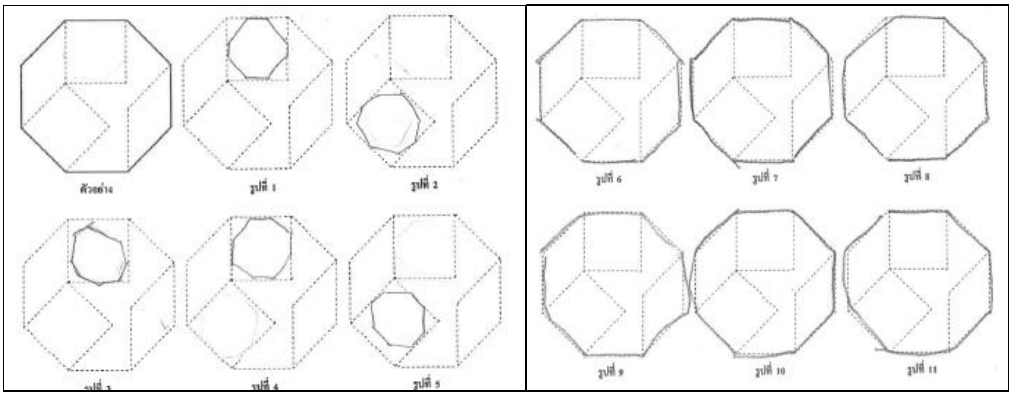


Figure 6. Students misunderstood the problem, and assumed that any octagon is a regular octagon.

In addition, we have calculated the frequency that each figure is spotted, as shown Figure 7. We point out that the third, sixth and ninth figures shown below are not only most often found in the students' answers, but also among the first few figures that the students drew in their answer sheets. We believe that problem-solving skills correlate strongly to observation skills, which seems to suggest that a large number of students either do not understand the questions or have difficulty observing an octagon inside the regular octagon. We hypothesise here that when the students were introduced to polygons in the classroom, the examples from the lesson were regular polygons, and teachers did not give enough examples of non-regular polygons. We posted the same problem and the students' statistics to three groups of math teachers for Grades 4-6 (approximately 150 in each group) at the SMTD teacher training workshops, and they too often found the sixth and ninth figures. When asked whether they had shown non-regular polygons in class, the resounding answer is "yes, but we spend more time on regular polygons."

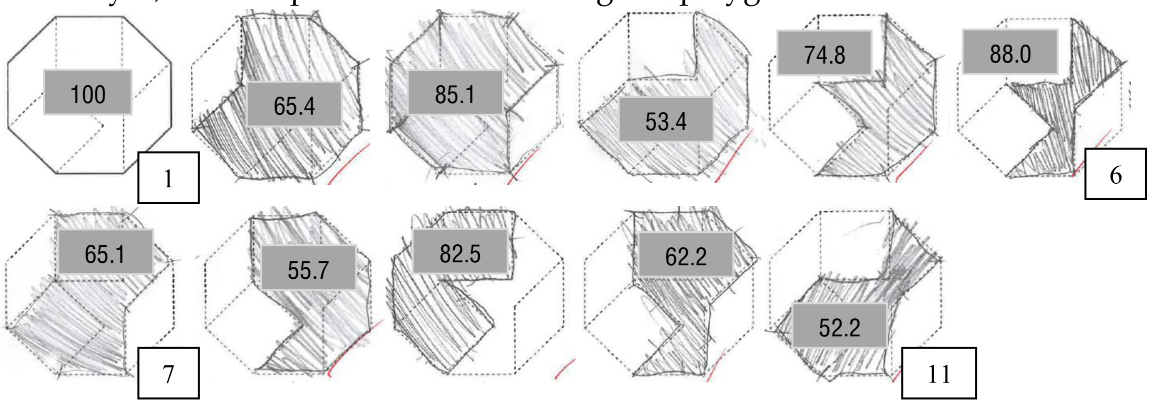


Figure 7. All possible solutions to Problem 10. The percentage number in each figure indicates of often the figure was discovered by the students.

Upper Level Results

Like in the lower primary level, we present problems translated to English and then the results of the students' performance as related to problem solving.

Problem 2: Given that a, b, c and d are four different prime numbers. Consider the following three numbers:

The first number is $a \times a \times a \times b \times b \times c \times c \times c \times c$

The second number is $a \times a \times b \times b \times b \times c \times c \times c \times d$

The third number is $a \times b \times b \times c \times c \times d$

If m denotes the biggest integer which can exactly divide all three numbers above, and

n denotes the smallest integer which can be exactly divided by all three numbers above, find the values m, n and $n \div m$. (IPST, 2013b)

Problem 3: Given that X4273Y is a six-digit number where X and Y are digits. If the six-digit number is divisible by 72, find the product of X and Y. Show your work. (IPST, 2013b)

Problem 7: Given that the point O is the centre of the half-circle which has a diameter of 20 centimetres long. Points M and N are the centres of two smaller half-circles as shown in the figure below. (IPST, 2013b)

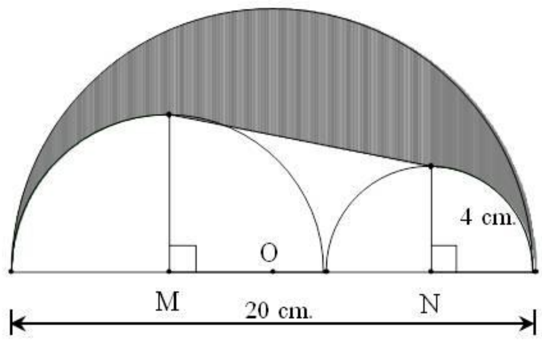


Figure 8. picture accompanies Problem 7 which asks to find the shaded area. The shaded region has area approximately how many square centimetres? (Use $\pi \approx 3.14$)

For the upper primary level, we focus Problems 2 and 3 on students' misconception in the respective concept, whereas Problem 7 is used to learn about students' strategic planning to solve the problem. In what follows, we provide details of our findings in each problem.

Problem 2: Result

In our analysis, we group the students into seven groups based on their mistakes because we want to understand their thinking and what hinders them from obtaining higher scores. Here we denote the usual abbreviations GCD and LCM for the greatest common divisors and least common multiples, respectively.

Table 4

Distribution of Scores on Problem 2 (Grades 4-6)

	Population Percent (%)	Group Description
I	35.0	Students obtain perfect scores, there is no mistake.
II	4.1	Students obtain correct answers for both GCD and LCM, but incorrect division.
III	17.9	Students obtain correct GCD, but incorrect LCM.
IV	3.4	Students obtain correct LCM, but incorrect GCD.
V	1.1	Students obtain incorrect answers for both GCD and LCM because their switch the meanings of GCD and LCM.

VI	27.6	Students obtain incorrect answers for both GCD and LCM because their do not know how to find GCD and LCM.
VII	10.9	Students do not attempt to solve this problem.

From Table 4, we observe that among the students who made mistakes on this problem, it is more likely that their mistake is on finding LCM than on finding GCD. Approximately 1% of students switch the meaning of LCM and GCD. However, there are a high number of students who do not know how to find both GCD and LCM in this problem. This could be because the problem is stated in terms of variables rather than numbers, and that are three of them. Among the common mistakes are: (i) students include of the factor d in GCD; (ii) students include the factor of d^2 in LCM; (iii) students perform short division to obtain LCM, but do not include the remainders in the calculation (i.e. they only multiply the quotients).

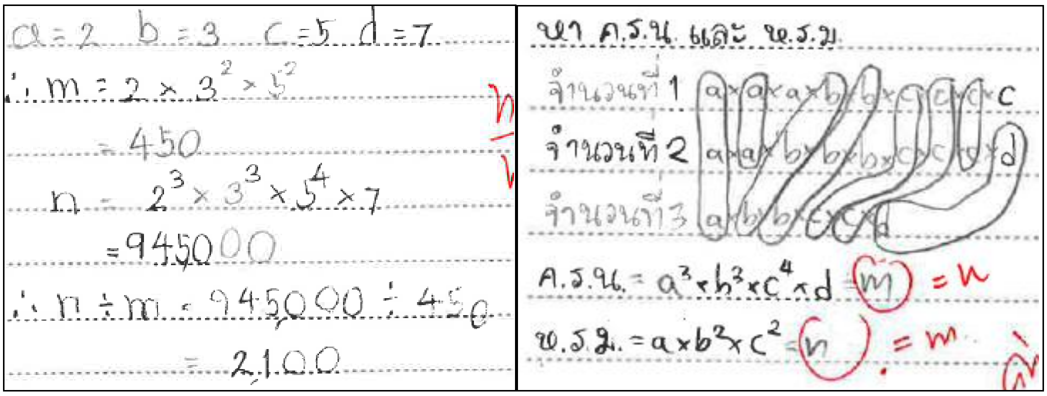


Figure 9. (left) student assigned the prime numbers to a, b, c and d. (right) the student switched the meaning of GCD and LCM.

We have found reasons to believe that this problem is difficult for students because it is written with symbols rather than numbers. Some students wrote $m = 1$ since a, b, c and d are prime factors. Many students replace variables with numerical values in their calculations (e.g. $a = 2$, $b = 3$, $c = 5$ and $d = 7$); see Figure 9. While this can indicate their understanding of prime factors, and such an approach can potentially give a correct answer, students who employed this approach fail to convert their answers back in terms of a, b, c and d.

Problem 3: Result

For this problem, we group the students into six groups according to how they obtained their answers. We also analyse their common mistakes. The statistical results are given in Table 5.

Table 5

Distribution of Scores on Problem 3 (Grades 4-6)

	Population Percent (%)	Group Description
I	9.3	Students used both divisibility properties by 9 and 8 (or 4) to obtain answers by solving equation.
II	7.4	Students used divisibility properties by 9 or by 8 and then use enumeration
III	15.2	Students used enumeration systematically, or “algorithmically.”
IV	13.7	Students used random enumeration.
V	6.7	Students guessed answer or did not show their work.
VI	47.7	Students did not attempt to solve this problem.

From Table 5, we can see that there are a high number of students who did not attempt to solve this problem, even though this problem is placed earlier on the exam. We hypothesise that maybe the students have read the problem, but it looks difficult to understand, or that they did not see an approach to solve this problem right away, so they worked on other problems first. We should emphasise that the grouping above does not rank the student scores. However, the students in Group IV and V tend to have lower scores (even when their answers are correct) because they did not show their work. In Groups II and III, students have skills to enumerate their answers systematically, but we cannot say the same about students in Group IV or V. Some students in Group V admitted that they guessed by trials and errors—by writing on their answer sheet “I do not know how to explain my answer; my points can be taken away.” See Figure 10 for examples of students’ solutions.

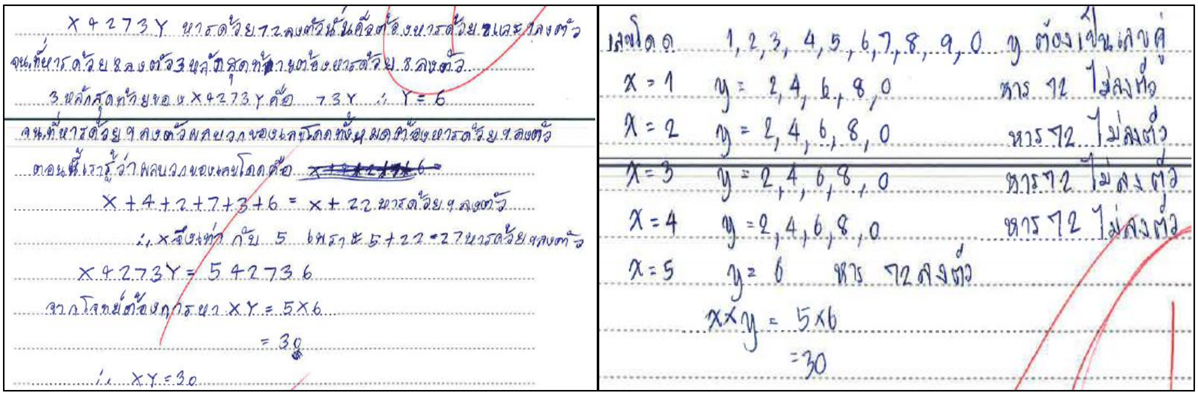


Figure 10. (left) a student solved systematically to obtain XY. (right) an enumeration approach employed by a student.

Qualitatively, we observe that the majority of students know the property of numbers divisible by 9 (that is: the sum of all digits in that number is divisible by 9), but relatively few students know the property of numbers divisible by 8, and are not able to find it. This problem is intended for the students to write the number as X4273Y = X×100,000 + 4×10,000 + 2×1,000 + 7×100 + 3×10 + Y and conclude that 7×100 + 3×10 + y must be divisible by 8. It is also common that the students used division by 4 rather than by 8 (e.g. 3×10 + Y must be divisible by 4), in which case it took them a little longer but they can obtain the final answer. Among the common trends are (i) students assumed that the digits must be distinct, and (ii) Y is assumed to be nonzero. It is unclear how these assumptions are obtained. For instance, did the student rule out Y = 0 because if Y = 0, then X×Y = 0?

What we can conclude from this problem is that (i) students know the proper of the numbers divisible by 9, (ii) students tend to solve the problems by enumerating possibilities in one way or the other (which can be a good start), and (iii) a large number of students attempt to solve this problem. It is debatable whether this problem can be used to measure students' creativity. We think that students did show their creativeness in their counting, for examples, those who used the fact that X + Y must have remainder 2 modulo 9 to enumerate their cases. However, it seems impossible to solve for X×Y without finding the values of X and Y individually.

Problem 7: Result

In this problem, the students are grouped into five groups according to their scores. The statistical results are shown in Table 6. We did not separate the students who did not attempt the problem from those who got zero score because there are very few, and from their answer sheets for this problem, it seems difficult to separate the two groups.

Table 6

Distribution of Scores on Problem 7 (Grades 4-6)

	Population Percent (%)	Group Description
I	40.7	Scores 10 points.
II	16.7	Scores range 8.5 – 9.5
III	19.4	Scores range 5.5 – 8.0
IV	6.4	Scores range 3.0 – 5.0
V	16.8	Scores range 0.0 – 2.5

Students in each group seem to make different mistakes (or perhaps, the grading scheme happens to group students with common mistakes in the same score range.). We will provide details for students' mistakes in each group. In Group I, there is no mistake to discuss, while in Group II, most mistakes are minor and numerical errors.

In Group III, it is evident that the students remembered the formula for finding the area of the circle as πr^2 but they did not necessarily know the meaning of r because many students used the value of the diameter, while others did not know how to find the value of a radius. Regrettably, we need not keep counts of how many students who made this mistake, but there seemed to account for a significant fraction of the population fraction. The main problem for the students in this group is either that they did not see the trapezoid or they did not know how to find the area of the trapezoid. This amounts for approximately 36.6% of students in this group. We believe that if they knew how to find the area of a trapezoid, they would have obtained the correct answer, i.e. that they had shown a correct strategy to find the shaded area, but missed key constituents.

In Group IV, we observe that the common mistakes are that (i) students knew the formula to find the area of the circle but they could not apply it (33.0%), that (ii) students did not see the relationship among the lengths of radii of the circles (25.4%), (iii) that students did not see the trapezoid or did not know how to find the area of the trapezoid (15.5%), and (iv) that the students misremembered the area formula for the circle (10.0%). Like in Group III, it is evident that the students knew how the shaded area is decomposed into smaller area, but they could not work out the answer (69.0%). We believe this indicates that students have the right strategic plan to solve the problem, but cannot execute the plan.

In Group V, we observe that many students did not know the formula, and that while some knew the formula to find the area of the circle, they could not apply it (20.7%). Again most students knew how the areas are decomposed to obtain the shaded area, but they could not work out the answer. In this group, it is also evident that the students misunderstood the term “approximate” in the problem statement. Some students tried to approximate the area by making unit squares inside the figure. Indeed, we ought to be careful with the word “approximate.” In this case, it is used such term because the approximated value of π is used, not because the problem instructs the students to estimate the area.

It is interesting to note that some students insisted on finding the area of the curvy region as shown in Figure 11 (left). Many of them were successful in doing so by calculating the area of the trapezoid, then subtracting it by a quarter of the area of two inner circles. This seems to suggest that the students did not want to sacrifice the previously obtained areas of the two half-circles, and could indicate their flaws in the planning stage of problem-solving.

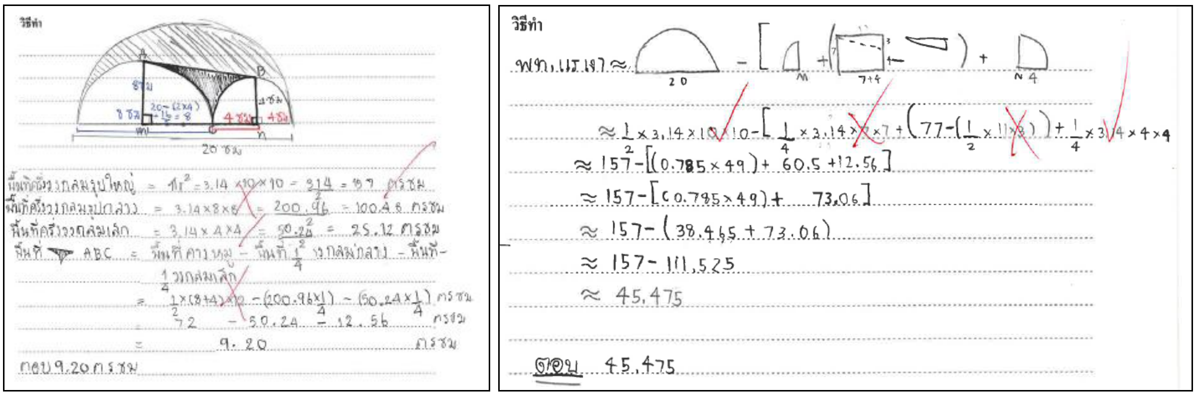


Figure 11. (left) a student insisted on solving for the dark-shaded area by finding the area of the trapezoid. (right) a student has a problem-solving plan, but cannot put all terms together.

Conclusion

From the second-round of the SMTD examination in Thailand, we have learned that even talented or above-average students often have misconceptions in mathematics. Their answers and explanation to open-ended exam questions in which they have to write out their reasons have shown their creativity and at the same time reflected what they have learned and memorized from their classrooms. Sometimes the students know mathematical facts, e.g. the formula for the area of a circle, very well but are not able to apply it to problem-solving. We have observed that students are able to perform calculations rather well, but their reasoning and problem-solving skills are far behind. In terms of measuring their creativity, some students are very creative in finding answers in mathematical pattern (e.g. Problem 8 from the lower primary level) and how they enumerate their cases to pinpoint the correct answer. However, we cannot make conclusion beyond this observation from the selected exam questions. In contrast, we can conclude that this group of students generally have a good plan for problem solving, but sometimes they are not able to execute the plan because they do not know how to find answers for constituent parts.

References

Bureau of Academics Affairs and Educations Standards, Ministry of Education. (2008). *Mathematics Core Curriculum and Standards,* Bangkok, Thailand. (in Thai)

Institute for the Promotion of Teaching Science and Technology. (2011). *Mathematics Textbooks for Primary Grades 1-6*. Bangkok, Thailand. Office of the Welfare Promotion Commission for Teachers and Educational Personnel. (in Thai)

Institute for the Promotion of Teaching Science and Technology. (2012a). *Mathematical Problem Solving Skills and Process*. (3rd Edition). Bangkok, Thailand. 3-Q Media Limited. (in Thai)

Institute for the Promotion of Teaching Science and Technology. (2012b). *Examples of Mathematics Achievement Evaluation*. Book 1. Bangkok, Thailand. 3-Q Media Limited. (in Thai)

Institute for the Promotion of Teaching Science and Technology. (2012c). *Measurement and Evaluation of Mathematics Achievement*. Bangkok, Thailand. SE-EdUCation. (in Thai)

Institute for the Promotion of Teaching Science and Technology. (2013a). *Examples of Mathematics Achievement Evaluation*. Book 2. Bangkok, Thailand. 3-Q Media Limited. (in Thai)

Institute for the Promotion of Teaching Science and Technology. (2013b). *SMTD Examination* (2013, July 30). Retrieved from http://genius.ipst.ac.th

Makanong, A. (2010). *Mathematical Problem Solving Skills: Development for Progress*. Bangkok, Thailand. Chulalongkorn University Press. (in Thai)

Polya, G. (1957). *How to solve it?* (2nd Edition). New York: Penguin Books.

Yeap, B. H. (2013). *Challenging Mathematics in Primary School National Examination in Singapore*. (July 30, 2013) Retrieved from www.amt.edu.ac/icmis16psinyeap.pdf

Author(s):
Ratchada YATRA, Institute for the Promotion of Teaching Science and Technology, Thailand
Email: ryatr@ipst.ac.th
Sujin SUWANNA, Department of Physics, Faculty of Science, Mahidol University, Thailand
Email: sujin.suw@mahidol.ac.th
Surat INTASANG, Faculty of Education, Ramkhamhaeng University, Thailand
Email: surat.intasang@gmail.com